A PowerPoint Presentation Package to Accompany

Applied Statistics in Business & Economics, 5th edition

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One-Sample Hypothesis Tests

Chapter Contents

9.1 Logic of Hypothesis Testing
9.2 Type I and Type II Error
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9.6 Testing a Proportion
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One-Sample Hypothesis Tests

Chapter 9

Chapter Learning Objectives (LO's)

LO9-1: Know the steps in testing hypotheses and define H_0 and H_1.

LO9-2: Define Type I error, Type II error, and power.

LO9-3: Formulate a null and alternative hypothesis for *μ or π.*

LO9-4: Explain decision rules, critical values, and rejection regions..

LO9-5: Perform a hypothesis test for a mean with known σ using z.

LO9-6: Use tables or Excel to find the p-value in tests of μ.

One-Sample Hypothesis Tests

Chapter Learning Objectives (LO's)

LO9-7: Perform a hypothesis test for a mean with unknown ***σ* using *t*.**

LO9-8: Perform a hypothesis test for a proportion and find the *p-value*

LO9-9: *Check whether normality may be assumed in testing a proportion.*

LO9-10: Interpret a power curve or OC curve (optional).

LO9-11: Perform a hypothesis test for a variance (optional).

LO9-1

9.1 The Logic of Hypothesis Testing

LO9-1: Know the steps in testing hypotheses and define H_0 and H_1.

- The process of **hypothesis testing** can be an iterative process, as illustrated in Figure 9.1.

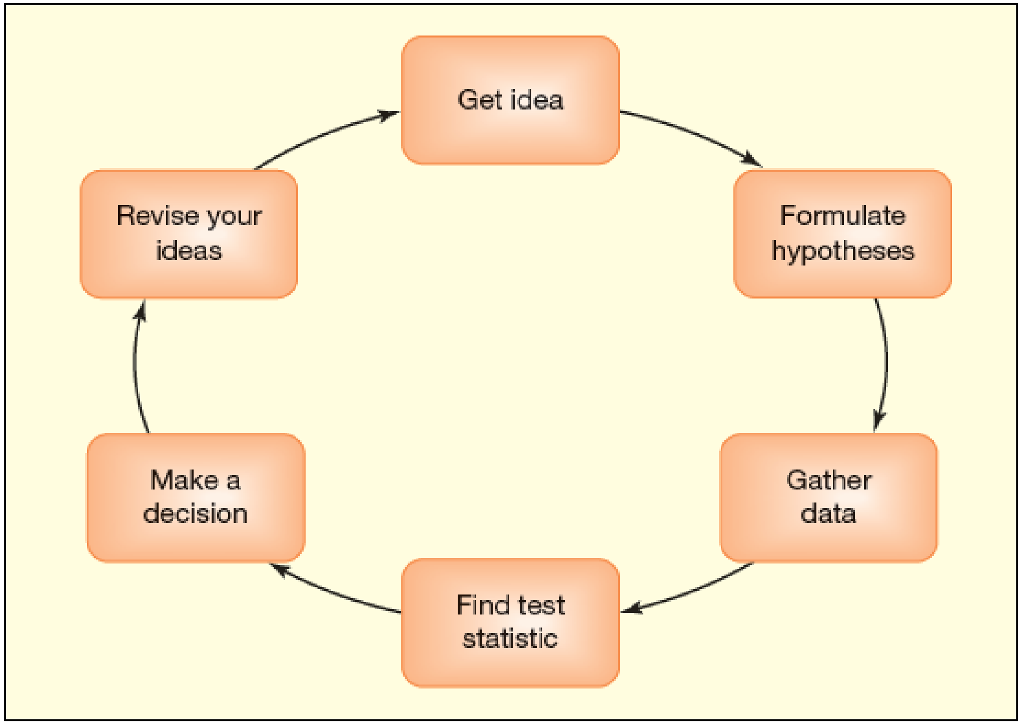


FIGURE 9.1

Hypothesis Testing as an Ongoing Process

LO9-1

9.1 The Logic of Hypothesis Testing

LO9-1: Know the steps in testing hypotheses and define H_0 and H_1.

Steps in Hypothesis Testing

Step 1: State the hypothesis to be tested.

Step 2: Specify what level of inconsistency with the data will lead to rejection of the hypothesis. This is called a *decision rule*.

Step 3: Collect data and calculate necessary statistics to test the hypothesis.

Step 4: Make a decision. Should the hypothesis be rejected or not?

Step 5: Take action based on the decision.

LO9-1

9.1 The Logic of Hypothesis Testing

LO9-1: Know the steps in testing hypotheses and define H_0 and H_1.

State the Hypothesis

- Hypotheses are a pair of mutually exclusive, collectively exhaustive statements about some fact about a population.
- One statement or the other must be true, but they cannot both be true.
- H_0: Null Hypothesis
  H_1: Alternative Hypothesis
- These two statements are hypotheses because the truth is unknown.

LO9-1

9.1 The Logic of Hypothesis Testing

State the Hypothesis

- Efforts will be made to reject the null hypothesis.
- If H_0 is rejected, we tentatively conclude H_1 to be the case.
- H_0 is sometimes called the *maintained hypothesis.*
- H_1 is called the *action alternative* because action may be required if we reject H_0 in favor of H_1.

Can Hypotheses be Proved?

- We cannot accept a null hypothesis, we can only *fail to reject* it.

Role of Evidence

- The null hypothesis is *assumed true* and a contradiction is sought.

LO9-2

9.2 Type I and Type II Error

LO9-3: Define Type I error, Type II error, and power.

Types of Error

- *Type I error*: Rejecting the null hypothesis when it is true. This occurs with probability α (level of significance).
- *Type II error*: Failure to reject the null hypothesis when it is false. This occurs with probability β.

	H_0 is true	***H_0 is false***
Reject H_0	Type I error	Correct decision
Fail to reject H_0	Correct decision	Type II error

LO9-2

9.2 Type I and Type II Error

Probability of Type I and Type II Errors

Key Term	*What Is It?*	*Symbol*	*Definition*	*Also Called*
Type I error	Reject a true hypothesis	α	$P(\text{reject } H_0 \mid H_0 \text{ is true})$	False positive
Type II error	Fail to reject a false hypothesis	β	$P(\text{fail to reject } H_0 \mid H_0 \text{ is false})$	False negative
Power	Correctly reject a false hypothesis	$1 - \beta$	$P(\text{reject } H_0 \mid H_0 \text{ is false})$	Sensitivity

- If we choose α = .05, we expect to commit a Type I error about 5 times in 100.
- β cannot be chosen in advance because it depends on α and the sample size.
- A small β is desirable, other things being equal.

LO9-2

9.2 Type I and Type II Error

Power of a Test

Power of a Test If the null hypothesis is false, we ought to reject it. If we do so, we have done the right thing, and there is no Type II error. The **power** of a test is the probability that a false hypothesis will be rejected, as it should be. More power is good, because power is the probability of doing the right thing. Power is the complement of beta risk ($1 - \beta$). If we have low β risk, we have high power:

Power

The probability of rejecting the null hypothesis when it is false is $1 - \beta$ and is called *power*. In medicine, the power of a test to correctly detect a disease is its *sensitivity*.

$$\text{Power} = P(\text{reject } H_0 \mid H_0 \text{ is false}) = 1 - \beta$$

- A low β risk means high power.
- Larger samples lead to increased power.

LO9-2

9.2 Type I and Type II Error

Relationship Between α and β

- Both a small α and a small β are desirable.
- For a given type of test and fixed sample size, there is a trade-off between α and β.
- The larger critical value needed to reduce α risk makes it harder to reject H_0, thereby increasing β risk.
- Both α and β can be reduced simultaneously only by increasing the sample size.

LO9-2

9.2 Type I and Type II Error

Consequences of Type I and Type II Errors

- The consequences of these two errors are quite different, and the costs are borne by different parties.
- *Example:* Type I error is convicting an innocent defendant, so the costs are borne by the defendant. Type II error is failing to convict a guilty defendant, so the costs are borne by society if the guilty person returns to the streets.
- Firms are increasingly wary of Type II error (failing to recall a product as soon as sample evidence begins to indicate potential problems.)

LO9-3

9.3 Decision Rules and Critical Values

LO9-3: Formulate a null and alternative hypothesis for *μ or π*.

- A *statistical hypothesis* is a statement about the value of a population parameter.
- A *hypothesis test* is a decision between two competing mutually exclusive and collectively exhaustive hypotheses about the value of the parameter.
- When testing a mean we can choose between three tests.

Left-Tailed Test	*Two-Tailed Test*	*Right-Tailed Test*
H_0: $\mu \geq \mu_0$	H_0: $\mu = \mu_0$	H_0: $\mu \leq \mu_0$
H_1: $\mu < \mu_0$	H_1: $\mu \neq \mu_0$	H_1: $\mu > \mu_0$

The application will dictate which of the three alternatives is appropriate. The *direction of the test* is indicated by which way the inequality symbol points in H_1:

LO9-3

9.3 Decision Rules and Critical Values

LO9-3: Formulate a null and alternative hypothesis for *μ or π*.

- The direction of the test is indicated by H_1:

  > indicates a right-tailed test
  < indicates a left-tailed test
  ≠ indicates a two-tailed test

Test Type	*Decision Rule*
Left-tailed	Reject H_0 if the test statistic < left-tail critical value
Two-tailed	Reject H_0 if the test statistic < left-tail critical value or if the test statistic > right-tail critical value
Right-tailed	Reject H_0 if the test statistic > right-tail critical value

Where Do We Get μ_0 (or π_0)?

The value of μ_0 (or π_0) that we are testing is a *benchmark* based on past experience, an industry standard, a target, or a product specification. The value of μ_0 (or π_0) does *not* come from a sample.

LO9-4

9.3 Decision Rules and Critical Values

LO9-4: Explain decision rules, critical values, and rejection regions.

Decision Rule

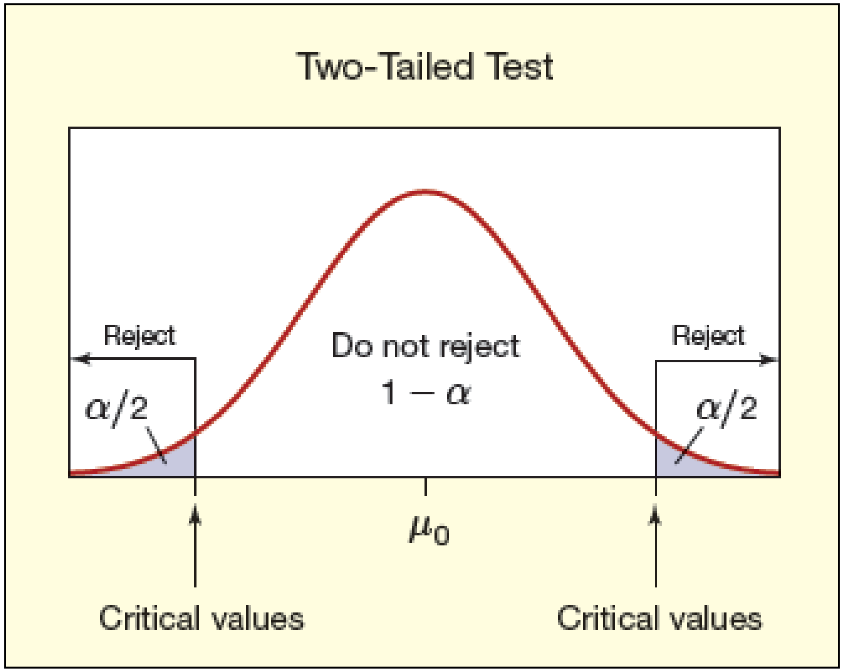
- A *test statistic* shows how far the sample estimate is from its expected value, in terms of its own standard error.
- The *decision rule* uses the known sampling distribution of the test statistic to establish the *critical value* that divides the sampling distribution into two regions.
- Reject H_0 if the test statistic lies in the *rejection region*.

LO9-4

9.3 Decision Rules and Critical Values

Decision Rule for Two-Tailed Test

- Reject H_0 if the test statistic < left-tail critical value or if the test statistic > right-tail critical value.



LO9-4

9.3 Decision Rules and Critical Values

When to use a One- or Two-Sided Test

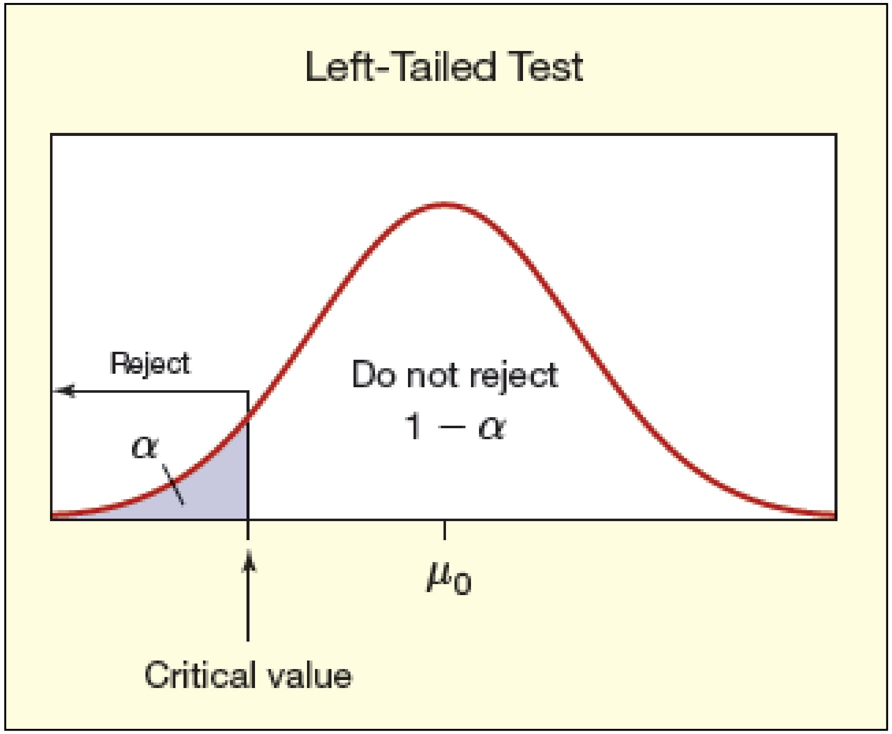
- A two-sided hypothesis test (≠) is used when direction (< or >) is of no interest to the decision maker
- A one-sided hypothesis test is used when
  - the consequences of rejecting H_0 are asymmetric, or
  - where one tail of the distribution is of special importance to the researcher.
- Rejection in a two-sided test guarantees rejection in a one-sided test, other things being equal.

LO9-4

9.3 Decision Rules and Critical Values

Decision Rule for Left-Tailed Test

- Reject H_0 if the test statistic < left-tail critical value.

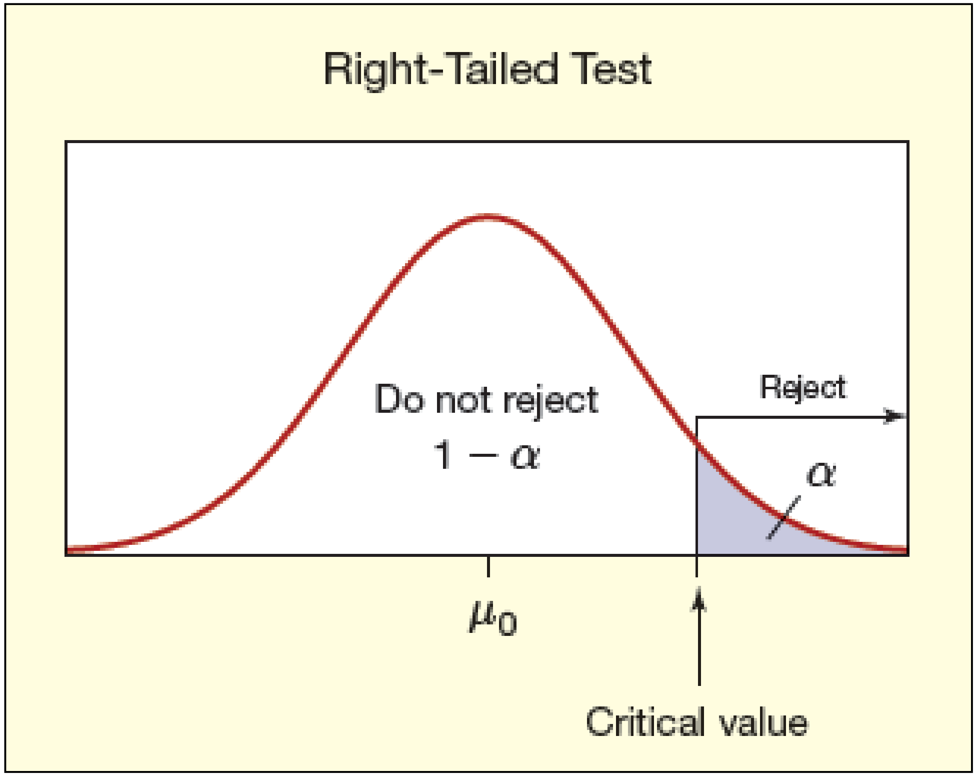


LO9-4

9.3 Decision Rules and Critical Values

Decision Rule for Right-Tailed Test

- Reject H_0 if the test statistic > right-tail critical value.



LO9-4

9.3 Decision Rules and Critical Values

Type I Error

- A reasonably small level of significance α is desirable, other things being equal.
- Chosen in advance, common choices for α are
  .10, .05, .025, .01 and .005
  (i.e., 10%, 5%, 2.5%, 1% and .5%).
- The α risk is the area under the tail(s) of the sampling distribution.
- In a two-sided test, the α risk is split with $\alpha/2$ in each tail since there are two ways to reject H_0.

LO9-5

9.4 Testing a Mean: Known Population Variance

LO9-5: *Perform a hypothesis test for a mean with known σ using z.*

- The hypothesized mean μ_0 that we are testing is a benchmark.
- The value of μ_0 does not come from a sample.
- The *test statistic* compares the sample mean with the hypothesized mean μ_0.

Test Statistic for a Mean: Known σ

Sample mean — Hypothesized mean

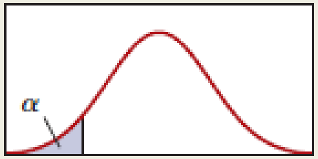
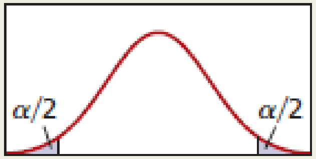
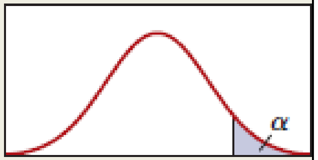
$$z_{calc} = \frac{\bar{x} - \mu_0}{\sigma_{\bar{x}}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

Standard error of the sample mean

LO9-5

9.4 Testing a Mean: Known Population Variance

- The test statistic is compared with a critical value from a table.
- The critical value is the boundary between two regions (reject H_0*, do not reject* H_0*) in the decision rule.*
- *The critical value* shows the range of values for the test statistic that would be expected by chance if the null hypothesis were true.

Level of Significance (α)	Left-Tailed Test 	Two-Tailed Test 	Right-Tailed Test 
.10	−1.282	±1.645	+1.282
.05	−1.645	±1.960	+1.645
.01	−2.326	±2.576	+2.326

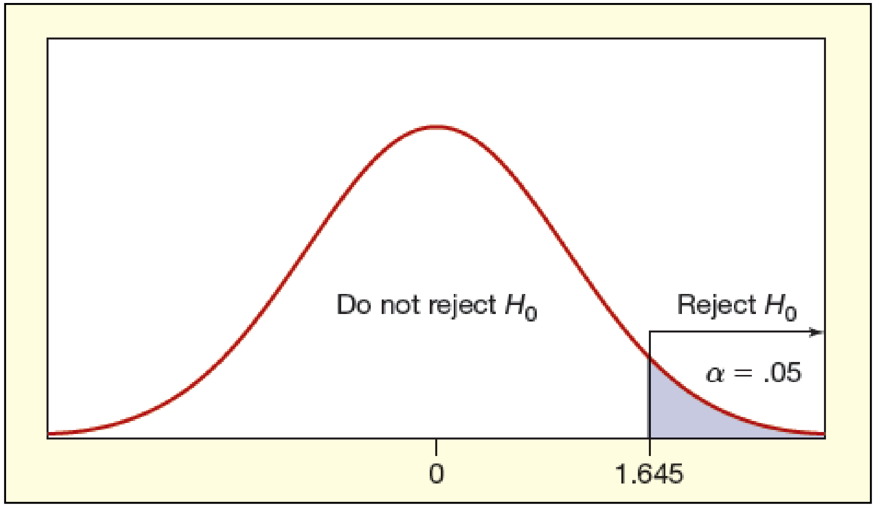
LO9-5

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing

Testing the Hypothesis

- Step 1: State the hypotheses. For example, H_0: $\mu \le 216$ mm
  H_1: $\mu > 216$ mm
- Step 2: Specify the decision rule. For example, for $\alpha = .05$ for the right-tail area, reject H_0 if $z_{calc} > 1.645$, otherwise do not reject H_0.



LO9-5

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing (continued):

Testing the Hypothesis

- Step 3: Collect Sample Data and Calculate the Test Statistic.
  If H_0 *is true,* then the test statistic should be near 0 because the sample mean *should be near* μ_0. *The value of the test* statistic is

$$z_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{216.0070 - 216.0000}{\frac{.0230}{\sqrt{50}}} = \frac{0.0070}{0.00325269} = 2.152$$

- Step 4: Make the decision.
  The test statistic falls in the right rejection region, so we reject the null hypothesis H_0: $\mu \leq 216$ *and conclude the alternative hypothesis* H_1: $\mu > 216$ at the 5% level of significance.

LO9-5

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing (continued):

Testing the Hypothesis

- Step 5: Take Action
  Now that we have concluded that the process is producing paper with an average width *greater than the specification, it is time to adjust the manufacturing process to* bring the average width back to specification. Our course of action could be to readjust the machine settings or it could be time to re-sharpen the cutting tools. At this point it is the responsibility of the process engineers to determine the best course of action.

LO9-6

9.4 Testing a Mean: Known Population Variance

LO9-6: Use Table or Excel to find the p-value in tests for μ.

p-Value Approach

- The *p*-value is the probability of the sample result (or one more extreme) assuming that H_0 is true.
- The *p*-value can be obtained using Excel's cumulative standard normal function.
- The *p*-value can also be obtained from Appendix C-2.
- Using the *p*-value, we reject H_0 if *p*-value $< \alpha$.

LO9-6

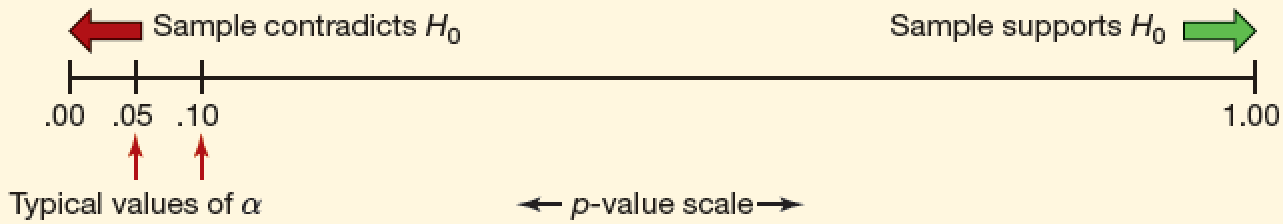
9.4 Testing a Mean: Known Population Variance

LO9-6: Use Table or Excel to find the p-value in tests for μ.

p-Value Approach

What Is a *p*-Value?

A sample statistic is a random variable that may differ from the hypothesized value merely by chance, so we do not expect the sample to agree *exactly* with H_0. The *p*-value is the probability of obtaining a test statistic as extreme as the one observed, assuming that the null hypothesis is true. A large *p*-value (near 1.00) tends to support H_0, while a small *p*-value (near 0.00) tends to contradict H_0. If the *p*-value is less than the chosen level of significance (α), then we conclude that the null hypothesis is false.



LO9-6

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing

Two-Tail Test of Hypothesis

Two-Tailed Test

In this case, the manufacturer decided that a two-tailed test would be more appropriate, since the objective is to detect a deviation from the desired mean in *either* direction.

Step 1: State the Hypotheses For a two-tailed test, the hypotheses are

H_0: $\mu = 216$ mm (product mean is what it is supposed to be)

H_1: $\mu \neq 216$ mm (product mean is not what it is supposed to be)

LO9-6

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing

Two-Tail Test of Hypothesis

Step 2: Specify the Decision Rule We will use the same $\alpha = .05$ as in the right-tailed test. But for a two-tailed test, we split the risk of Type I error by putting $\alpha/2 = .05/2 = .025$ in each tail. For $\alpha = .05$ in a two-tailed test, the critical value is $z_{.025} = \pm 1.96$ so the decision rule is

Reject H_0 if $z_{\text{calc}} > +1.96$ or if $z_{\text{calc}} < -1.96$

Otherwise do not reject H_0

LO9-6

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing

Two-Tail Test of Hypothesis

Step 3: Calculate the Test Statistic The test statistic is *unaffected by the hypotheses or the level of significance*. The value of the test statistic is the same as for the one-tailed test:

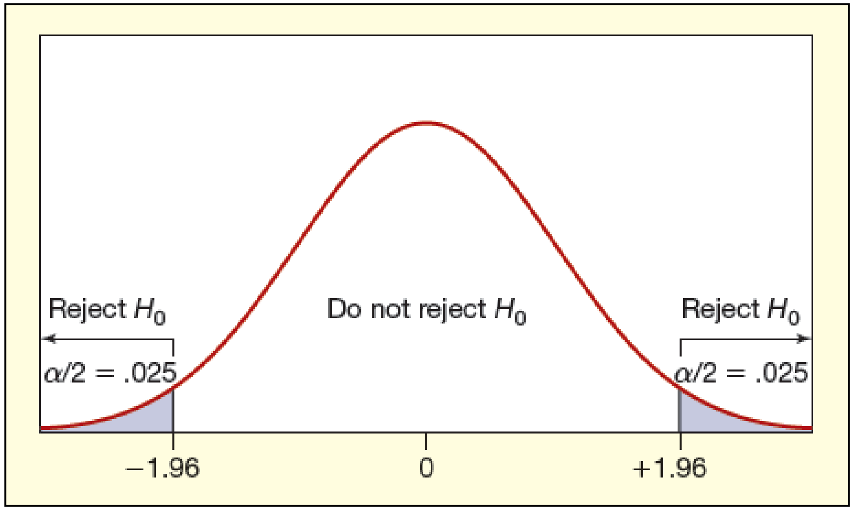
$$z_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}} = \frac{216.0070 - 216.0000}{\frac{.0230}{\sqrt{50}}} = \frac{.0070}{.00325269} = 2.152$$

LO9-6

9.4 Testing a Mean: Known Population Variance

Example: Paper Manufacturing

Two-Tail Test of Hypothesis



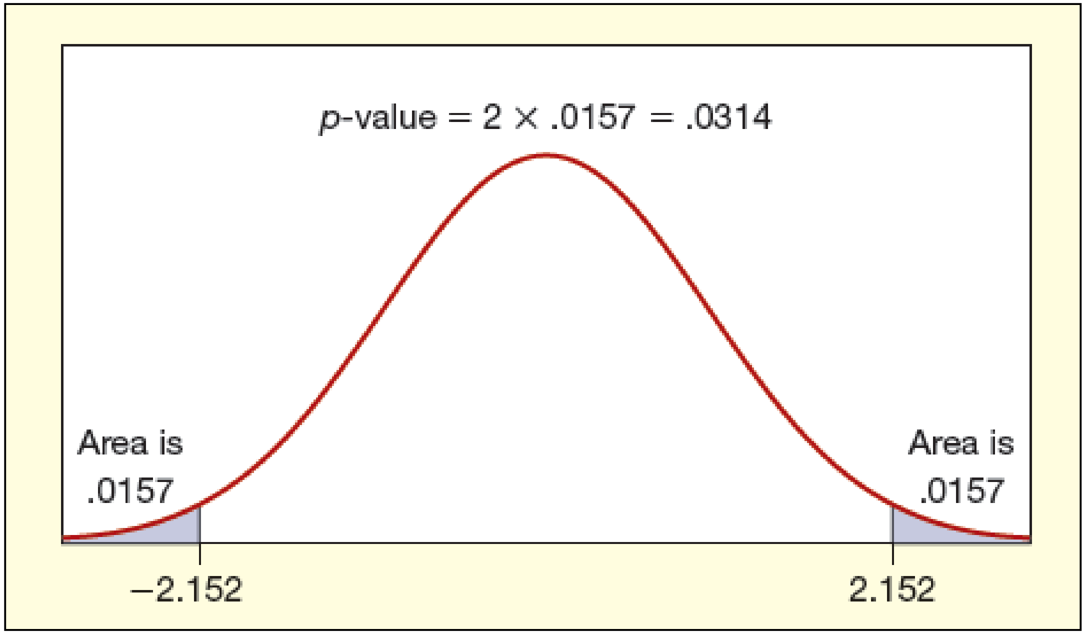
Step 4: Make the Decision Since the test statistic falls in the right tail of the rejection region, we reject the null hypothesis H_0: $\mu = 216$ and conclude H_1: $\mu \neq 216$ at the 5 percent level of significance. Another way to say this is that the sample mean *differs significantly* from the desired specification at $\alpha = .05$ in a two-tailed test. Note that this decision is rather a close one, since the test statistic just barely falls into the rejection region.

LO9-6

9.4 Testing a Mean: Known Population Variance

Testing the Hypothesis

Using the p-Value Approach



P-value = 0.0314 < α = 0.05, so the null hypothesis is rejected.

LO9-6

9.4 Testing a Mean: Known Population Variance

Analogy to Confidence Intervals

- A two-tailed hypothesis test at the 5% level of significance (α = .05) is exactly equivalent to asking whether the 95% confidence interval for the mean includes the hypothesized mean.
- If the confidence interval includes the hypothesized mean, then we cannot reject the null hypothesis.

LO9-7

9.5 Testing a Mean: Unknown Population Variance

LO9-7: Perform a hypothesis test for a mean with unknown σ using *t*.

Using Student's t

- When the population standard deviation σ is unknown and the population may be assumed normal, the test statistic follows the Student's *t* distribution with $n - 1$ degrees of freedom.

Test Statistic for a Mean: σ Unknown

Sample mean Hypothesized mean

$$t_{calc} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} \quad \text{if } \sigma \text{ is unknown} \qquad (9.2)$$

Sample st. dev.

LO9-7

9.5 Testing a Mean: Unknown Population Variance

Example: Hot Chocolate

Testing the Hypothesis

- Step 1: State the hypotheses
  H_0: $\mu = 142$
  H_1: $\mu \neq 142$
- Step 2: Specify the decision rule - for $\alpha = .10$ for the two-tail test and with d.f. n – 1 = 24-1 = 23, reject H_0 if $t_{calc} > 1.714$ or if $t_{calc} < -1.714$, otherwise do not reject H_0.

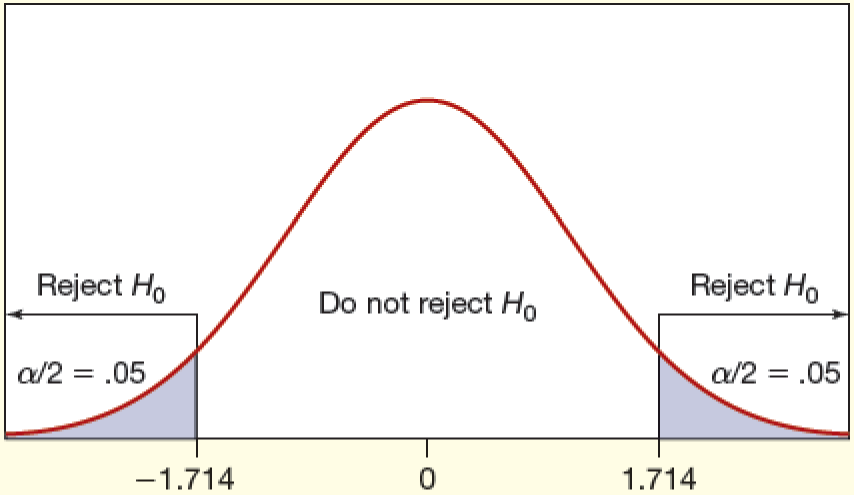
LO9-7

9.5 Testing a Mean: Unknown Population Variance

Example: Hot Chocolate

Testing the Hypothesis

FIGURE 9.8 Two-Tailed Test for a Mean Using t for $d.f. = 23$



Critical regions for the test.

LO9-7

9.5 Testing a Mean: Unknown Population Variance

Example: Hot Chocolate

Testing the Hypothesis

- Step 3: Collect Sample Data and Calculate the Test Statistic If H_0 *is true,* then the test statistic should be near 0 because the sample mean *should be near* μ_0. *The value of the test* statistic is

$$t_{\text{calc}} = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}} = \frac{141.375 - 142}{\frac{1.99592}{\sqrt{24}}} = \frac{-.6250}{.40742} = -1.534$$

LO9-7

9.5 Testing a Mean: Unknown Population Variance

Example: Hot Chocolate

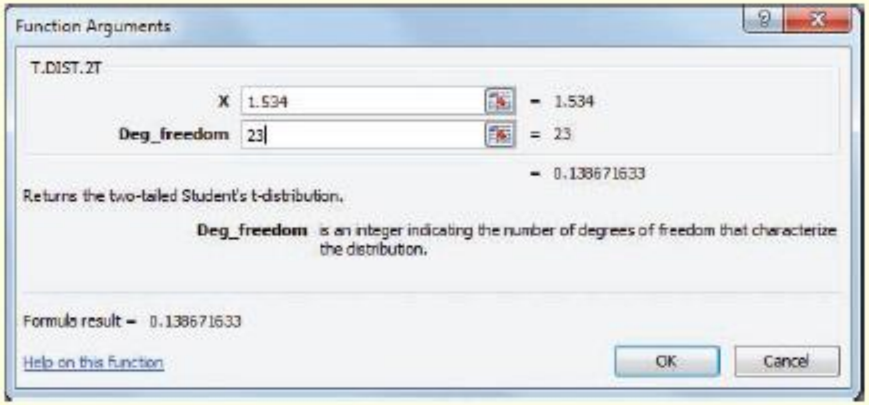
Testing the Hypothesis

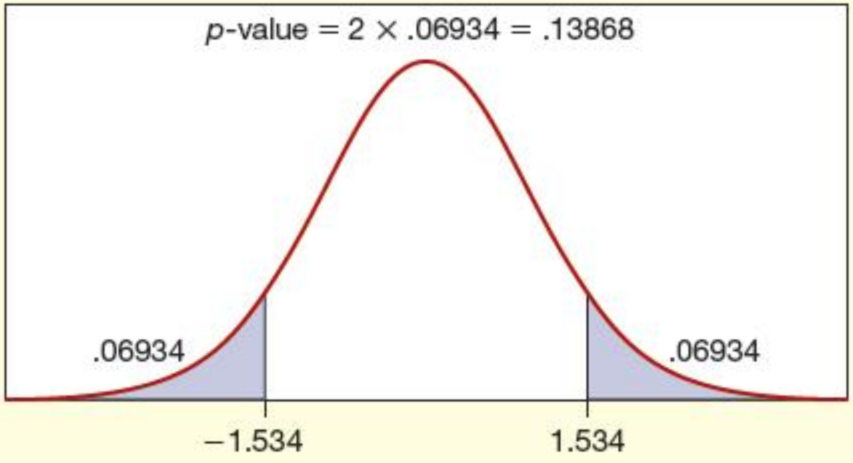
- Step 4: Since the test statistic lies within the range of chance variation, we cannot reject the null hypothesis $H_0: \mu = 142$.
- *Note:* The p-value = .13867. Since .13867 > .1 (level of significance), do not reject the null hypothesis.

LO9-7

9.5 Testing a Mean: Unknown Population Variance

Using the *p*-value





LO9-7

9.5 Testing a Mean: Unknown Population Variance

Confidence Intervals versus Hypothesis Test

- A two-tailed hypothesis test at the 10% level of significance (α = .10) is equivalent to a two-sided 90% confidence interval for the mean.
- If the confidence interval does not include the hypothesized mean, then we reject the null hypothesis.
- The 90% confidence interval for the mean is given next.

$$\bar{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}} \quad \text{or} \quad 141.375 \pm (1.714)\frac{1.99592}{\sqrt{24}} \quad \text{or} \quad 141.375 \pm .6983$$

- Since μ = 142 lies within the 90 percent confidence interval [140.677, 142.073], we cannot reject the hypothesis H0: μ = 142 at α = .10 in a two-tailed test.

LO9-8

9.6 Testing a Proportion

LO9-8: Perform a hypothesis test for a proportion and find the *p*-value.

- To conduct a hypothesis test, we need to know
  - the parameter being tested
  - the sample statistic
  - the sampling distribution of the sample statistic
- The sampling distribution tells us which test statistic to use.
- A sample proportion *p* estimates the population proportion π.
- Remember that for a large sample, *p* can be assumed to follow a normal distribution. If so, the test statistic is *z*.

LO9-8, 9 9.6 Testing a Proportion

LO9-9: Check whether normality may be assumed in testing a proportion.

If we can assume a normal sampling distribution, then the test statistic would be the z-score. Recall that the sample proportion is

$$p = \frac{x}{n} = \frac{\text{number of successes}}{\text{sample size}}$$

The test statistic, calculated from sample data, is the difference between the sample proportion p and the hypothesized proportion π_0 divided by the *standard error of the proportion* (sometimes denoted σ_p):

Test Statistic for a Proportion

$$z_{\text{calc}} = \frac{p - \pi_0}{\sigma_p} = \frac{p - \pi_0}{\sqrt{\dfrac{\pi_0(1 - \pi_0)}{n}}}$$

The value of π_0 we are testing is a **benchmark**, such as past performance, an industry standard, or a product specification. The value of π_0 does *not* come from a sample.

NOTE: A rule of thumb to assume normality is if $n\pi_0 \geq 10$ and $n(1 - \pi_0) \geq 10$.

LO9-8, 9

9.6 Testing a Proportion

- The value of π_0 that we are testing is a *benchmark* such as past experience, an industry standard, or a product specification.
- The value of π_0 does not come from a sample.

Left-Tailed Test	*Two-Tailed Test*	*Right-Tailed Test*
$H_0\colon \pi = \pi_0$	$H_0\colon \pi = \pi_0$	$H_0\colon \pi = \pi_0$
$H_1\colon \pi < \pi_0$	$H_1\colon \pi \neq \pi_0$	$H_1\colon \pi > \pi_0$

LO9-8, 9

9.6 Testing a Proportion

Critical Value

- The test statistic is compared with a *critical z value* from a table.
- The critical value shows the range of values for the test statistic that would be expected by chance if the H_0 were true.

Example: Return Policy

Steps in Testing a Proportion

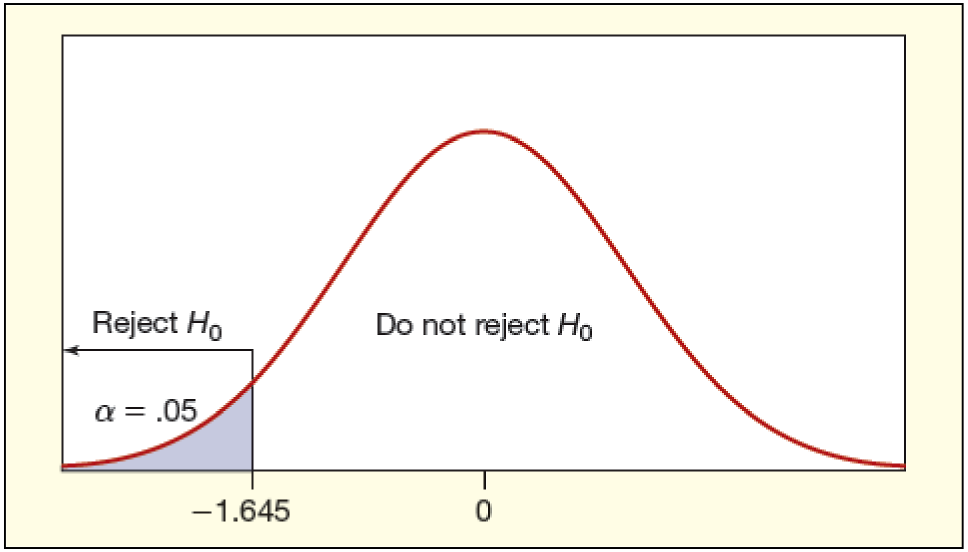
- Step 1: State the hypotheses
  For example, H_0: $\pi \geq .13$
  H_1: $\pi < .13$

LO9-8, 9

9.6 Testing a Proportion

Steps in Testing a Proportion

- Step 2: Specify the decision rule
  For $\alpha = .05$
  for a left-tail area,
  reject H_0 if $z < -1.645$,
  otherwise do not
  reject H_0.



LO9-8, 9

9.6 Testing a Proportion

Steps in Testing a Proportion

- Step 3: Collect Sample Data and Calculate the Test Statistic
  If *H_0 is true,* then the test statistic should be near 0 because the sample mean *should be near μ_0. The value of the test* statistic is given next.

$$z_{\text{calc}} = \frac{p - \pi_0}{\sqrt{\dfrac{\pi_0(1-\pi_0)}{n}}} = \frac{.088 - .13}{\sqrt{\dfrac{.13(1-.13)}{250}}} = \frac{-.042}{.02127} = -1.975$$

- Step 4: Since the test statistic lies in the left-tail rejection region, we reject the null hypothesis H_0: $\pi \geq .13$.

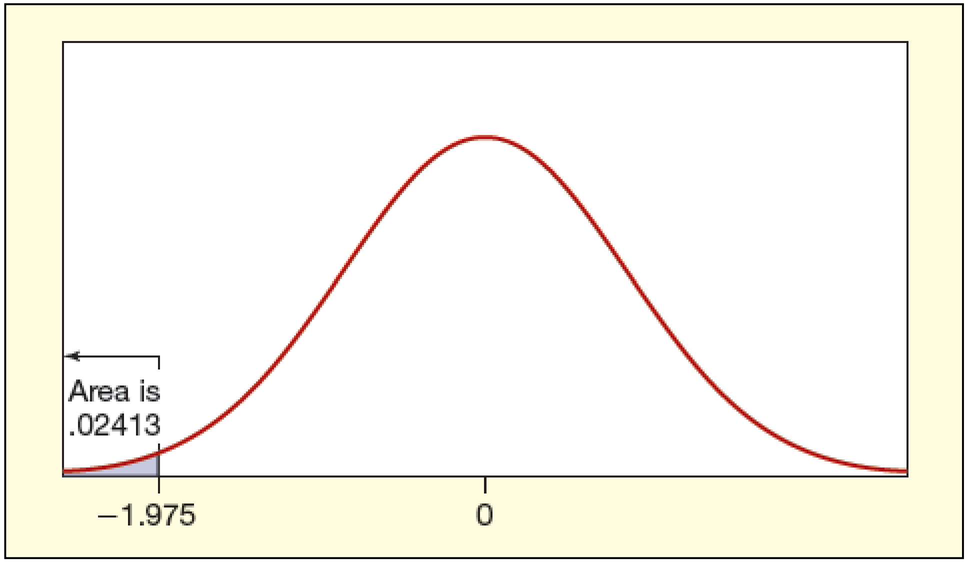
LO9-8, 9

9.6 Testing a Proportion

Calculating the p-Value

FIGURE 9.13

p-Value for a Left-Tailed Test with $z_{calc} = -1.975$



The *smaller* the *p*-value, the more we want to *reject* H_0. Does this seem backward? You might think a large *p*-value would be "more significant" than a small one. But the *p*-value is a direct measure of the level of significance at which we could reject H_0, so *a smaller* p-*value is more convincing*. For the left-tailed test, the *p*-value tells us that there is a .02413 probability of getting a sample proportion of .088 or less if the true proportion is .13; that is, such a sample would arise by chance only about 24 times in 1,000 tests if the null hypothesis is true. In our left-tailed test, we would reject H_0 because the *p*-value (.02413) is smaller than α (.05). In fact, we could reject H_0 at *any* α greater than .02413.

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9.6 Testing a Proportion

The effect of α

Would the decision be the same if we had used a different level of significance? Table 9.7 shows some possibilities. *The test statistic is the same regardless of* α. While we can reject the null hypothesis at $\alpha = .10$ or $\alpha = .05$, we cannot reject at $\alpha = .01$. Therefore, we would say that the current return rate differs from the historical return rate at the 10 percent and 5 percent levels of significance, but not at the 1 percent level of significance.

α	*Test Statistic*	*Two-Tailed Critical Values*	*Decision*
.10	$z_{calc} = -1.975$	$z_{.05} = \pm 1.645$	Reject H_0
.05	$z_{calc} = -1.975$	$z_{.025} = \pm 1.960$	Reject H_0
.01	$z_{calc} = -1.975$	$z_{.005} = \pm 2.576$	Don't reject H_0

TABLE 9.6

Effect of Varying α

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9.6 Testing a Proportion

The effect of α

Which level of significance is the "right" one? They all are. It depends on how much Type I error we are willing to allow. Before concluding that $\alpha = .01$ is "better" than the others because it allows less Type I error, you should remember that smaller Type I error leads to increased Type II error. In this case, Type I error would imply that there has been a change in return rates when in reality nothing has changed, while Type II error implies that the software had no effect on the return rate, when in reality the software did decrease the return rate.

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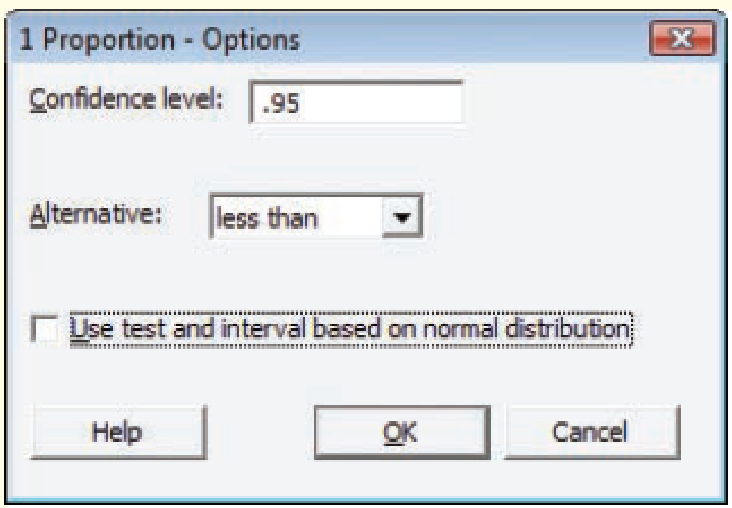
9.6 Testing a Proportion

Small Samples and Non-Normality

- In the case where $n\pi_0 < 10$, use MINITAB (or any other appropriate software) to test the hypotheses by finding the exact binomial probability of a sample proportion *p*.
  For example,

FIGURE 9.17

MINITAB Small-Sample Test of a Proportion



Test and CI for One Proportion

Test of p = 0.125 vs p < 0.125

Sample	X	N	Sample p	95% Upper Bound	Exact P-Value
1	1	16	0.062500	0.263957	0.388

LO9-10

9.7 Power Curves and OC Curves (Optional)

LO9-10: Interpret a power curve or OC curve (optional).

Refer to the text for this section.

LO9-11

9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Hypothesis Test for a Single Population Variance.

Null Hypothesis: H_0: $\sigma^2 = \sigma_0{}^2$

Alternative Hypothesis:

H_1: $\sigma^2 > \sigma_0{}^2$ H_1: $\sigma^2 < \sigma_0{}^2$ H_1: $\sigma^2 \neq \sigma_0{}^2$

Test Statistic: χ^2(calculated) = $\frac{(n-1)s^2}{\sigma_0{}^2}$

LO9-11 9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Hypothesis Test for a Single Population Variance.

Decision Rule:

1. Right–tail test: Reject H_0 if χ^2(calculated) > χ^2(upper).
2. Left–tail test: Reject H_0 if χ^2(calculated) < χ^2(lower).
3. Two-tail test: Reject H_0 if χ^2(calculated) < χ^2(lower) or if Reject H_0 if χ^2(calculated) > χ^2(upper).

LO9-11

9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Example: Attachment Times

Historical statistics show that the standard deviation of attachment times for an instrument panel in an automotive assembly line is $\sigma = 7$ seconds. Observations on 20 randomly chosen attachment times are shown in Table 9.11 (see the next slide). At $\alpha = .05$, does the variance in attachment times differ from the historical variance ($\sigma^2 = 7^2 = 49$)?

LO9-11

9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Example: Attachment Times

TABLE 9.11 Panel Attachment Times (seconds) Attachment

120	143	136	126	122
140	133	133	131	131
129	128	131	123	119
135	137	134	115	122

LO9-11

9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Example: Attachment Times

The sample mean is $\bar{x} = 129.400$ with a standard deviation of $s = 7.44382$. We ignore the sample mean since it is irrelevant to this test. For a two-tailed test, the hypotheses are

$$H_0: \sigma^2 = 49$$
$$H_1: \sigma^2 \neq 49$$

For a test of one variance, assuming a normal population, the test statistic follows the **chi-square distribution** with degrees of freedom equal to $d.f. = n - 1 = 20 - 1 = 19$. Denoting the hypothesized variance as σ_0^2, the test statistic is

$$\chi^2_{\text{calc}} = \frac{(n-1)s^2}{\sigma_0^2} \qquad \text{(test for one variance)}$$

$$\chi^2_{\text{calc}} = \frac{(n-1)s^2}{\sigma^2} = \frac{(20-1)(7.44382)^2}{7^2} = 21.49$$

LO9-11

9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Example: Attachment Times

Reject H_0 if $\chi^2_{\text{calc}} < \chi^2_{\text{lower}}$ or if $\chi^2_{\text{calc}} > \chi^2_{\text{upper}}$

Otherwise do not reject H_0

We can use the Excel function =CHISQ.INV to get the critical values:

$$\chi^2_{\text{lower}} = \text{CHISQ.INV}(\alpha/2, d.f.) = \text{CHISQ.INV}(0.025,19) = 8.907$$

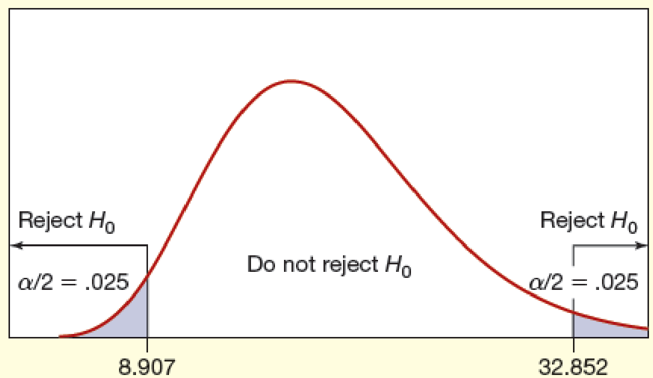
$$\chi^2_{\text{upper}} = \text{CHISQ.INV}(1-\alpha/2, d.f.) = \text{CHISQ.INV}(0.975,19) = 32.852$$

LO9-11 9.8 Tests for One Variance (Optional)

LO9-11: Perform a hypothesis test fro a variance (optional).

Example: Attachment Times

FIGURE 9.25 Decision Rule for Chi-Square Test



Because the test statistic is within the middle range, we conclude that the population variance does not differ significantly from 49; that is, the assembly process variance is unchanged.